Answer #181

The answer is (a): the arm will rotate clockwise immediately when the \textit{ac} power is applied, as seen in an mpeg video by clicking your mouse on the photograph below.

![Image of a Van de Graaff generator](image)

This device is often assumed to work like a rocket - that is, the electrons leave the points of the pinwheel like a rocket's exhaust, causing a reaction force on the rotor that gives it come acceleration. This is incorrect!

Let's go back and analyze the motion of the pinwheel on top of the Van de Graaff generator. The magnitude of the angular acceleration of the pinwheel on the van de Graaff generator can be estimated THE WRONG WAY as follows:

Let's assume that the device works like a rocket, with the torque on the pinwheel provided by the reaction force of electrons shot off the points at the end of the arms.

The angular acceleration \( \alpha \) is equal to the torque divided by the moment of inertia, or

\[
a = \frac{T}{I} = \frac{RF}{I},
\]

where

- \( R \) = the radius of the arm (about 10 cm or 0.1 m)
- \( I \) = the moment of inertia (three arms, total of 13 gm)

for a total \( I = MR^2/3 = 5 \times 10^{-5} \text{ kg m}^2 \)

and the force \( F \) on the point at radius \( R \) at the end of the pinwheel is

(number of electrons per second) x (momentum of each electron after it leaves the point)

so \( F = (m/t) \times v \)

where \( m = m_e \times I_{vdg}/q_e \)
and $v_e^2 = 2 m_e K_e$

where $K_e$ is the kinetic energy of each electron as it leaves the point.

Approximating the kinetic energy as about 100keV (dome voltage 100 kV), and the van de Graaff current as about 100 microamperes, the angular acceleration comes out to be about $10^{-3}$ sec$^{-2}$.

Now let's look at how fast the pinwheel accelerates in the video. It appears to accelerate about one radian (one-sixth of a rotation) in a second or so. Using this approximate value for the experimental angular acceleration of about 1 sec$^{-2}$, around one radian per second per second, the difference is several orders of magnitude.

There must therefore be another more accurate explanation for this phenomenon, and there is.

When the electrons are ejected from the points on the pinwheel, they immediately attach themselves to atoms of gas in the air next to the point, creating a cloud of negatively charged gas adjacent to the negatively charged point on the pinwheel. There is then an electrostatic force between the point on the pinwheel and the relatively massive charged cloud of gas adjacent to it. This electrostatic force is sufficient to accelerate the pinwheel, as the recoil force acts on the relatively massive charged gas cloud rather than the virtually massless electrons. It is left as an exercise for the student to make an approximate calculation of this effect and to verify the calculations presented above.