## Answer #290

**Part 1:** The answer is (a): two polaroids, with their axes diagonal, although which is the diagonal at  $+45^{\circ}$  (counterclockwise) from the vertical and which is the diagonal at  $-45^{\circ}$  (clockwise) from the vertical cannot be determined. We will arbitrarily represent the  $+45^{\circ}$  (clockwise) direction by the *upward* arrow and the  $-45^{\circ}$  (counterclockwise) direction by the *downward* arrow. Click on the photograph at the left below to see what happens when a polaroid is rotated in front of or behind the "up arrow" cube. Click on the photograph at the center below to see what happens when the two cubes are rotated about the optic axis.



**Part 2:** The answer is (d) a combination of three of the individual optical elements. The side with the up arrow is a polaroid at the  $+45^{\circ}$  (clockwise) angle; the side with the down arrow is a polaroid at the  $-45^{\circ}$  (counterclockwise) angle. Sandwiched between these two polaroids is a quarter-wave plate, aligned with its "extraordinary" axis aligned vertically. This will delay the extraordinary ray by 90° with respect to the ordinary ray, temporarily creating circularly polarized light. At the time the light leaves the cube it will have rotated by 90° in the either the clockwise or the counterclockwise direction, where it is linearly polarized by a polarizer oriented along the opposite diagonal.

These blocks can be used to represent the quantum mechanical "spin up" and "spin down" states of the electron, where the block with the quarter wave plate between the two crossed polaroids would represent the quantum mechanical "spin flip" operator, changing the electron state from spin up to spin down. Written in matrix form:

$$\begin{aligned} Q_{UAN} T_{UM} & \text{STATES}: \\ |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} & (\text{SPIN } UP) \\ |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} & (\text{SPIN } DOWN) \\ OPERATORS: \\ |\uparrow\rangle\langle\uparrow| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} & (UP) \\ |\downarrow\rangle\langle\downarrow| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} & (DOWN) \\ |\downarrow\rangle\langle\downarrow| = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} & (UP \to DOWN) \\ |\uparrow\rangle\langle\downarrow| = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} & (DOWN \to UP) \end{aligned}$$

For example, applying the up-down spin flip operator to the spin-up state will flip the spin, resulting in a spin-down state. The final state is one with polarization angle of -45°, so polarized light is seen when you look through the sequence of two cubes. Rotating a polaroid adjacent to either cube demonstrates that the light is polarized and/or the cube acts as a polarization analyzer.

**Part 3.** The difference in color is due to the effect of the quarter-wave plate. The light coming into the up-down / down-up "spin flip" operator is polarized, then passed through the quarter-wave plate. The quarter-wave plate is the thickness required to change the phase of *yellow* light by a quarter of a wavelength. However, the (negative) yellow filter allows a range of light to pass, because it removes only blue, the complementary color to yellow in the spectrum. The yellow is rotated properly by the quarter-wave plate, and to a lesser extent so are the neighboring red and green wavelengths. However, some of the deeper green wavelengths are not quite phase shifted by a quarter wavelength, so they pass through the polarizer on the other side of the quarter-wave plate. What should be "no light" becomes "green."

Question #291 is a follow-up to this question.



For questions and comments regarding the *Question of the Week* contact <u>Dr. Richard E. Berg</u> by e-mail or using phone number or regular mail address given on the <u>Lecture-Demonstration Home Page</u>.