Answer #297

The answers are:

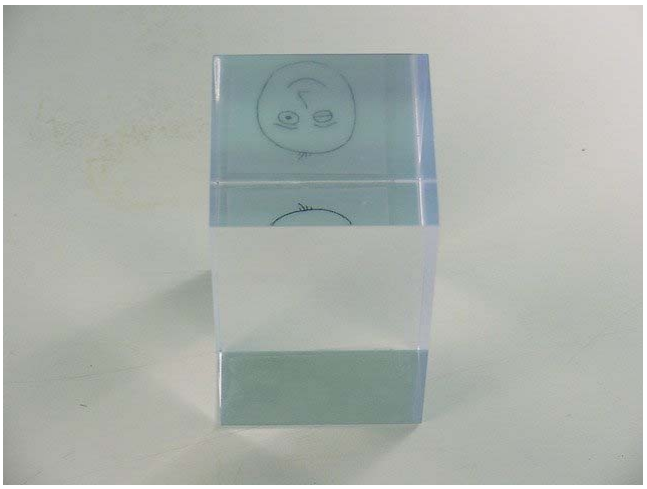
	View "look"
(1)	(a)
(2)	(e)
(3)	(b)
(4)	(b)
(5)	(e)
(6)	(e)
(7)	(c)
(8)	(c)
(9)	(e)

You can see these situations in the sequence of photographs below. Each photograph contains two views: the view through the side closest to the camera and the view of the internal reflection off the side opposite the camera. Notice that you cannot see *any* likeness of OTTO through the sides of the plastic block. A very elegant proof of this has been derived by John Roeder in *The Physics Teacher*, Volume 45 (March 2007), page 182, in an excellent short article called *Why Light Won't Refract Through Adjacent Faces of a Cube*, PDF or Word format.

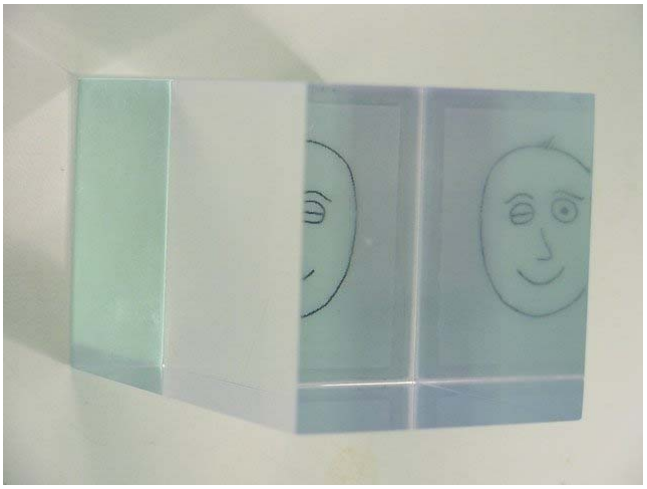
You can see a reflection of OTTO off the opposite internal side of the block, as seen in each photograph. Note that these internal reflections are *inverted front-to-back*, with each inner surface acting as a plane mirror. See Question of the Week #18 (Archive 1) for a more detailed discussion of reflection from a plane mirror. Each of the pictures showing the internal reflection of OTTO also show a bit of the direct downward view for comparison. The direct downward view of OTTO, called "look (a), is reproduced below.

OTTO direct view 

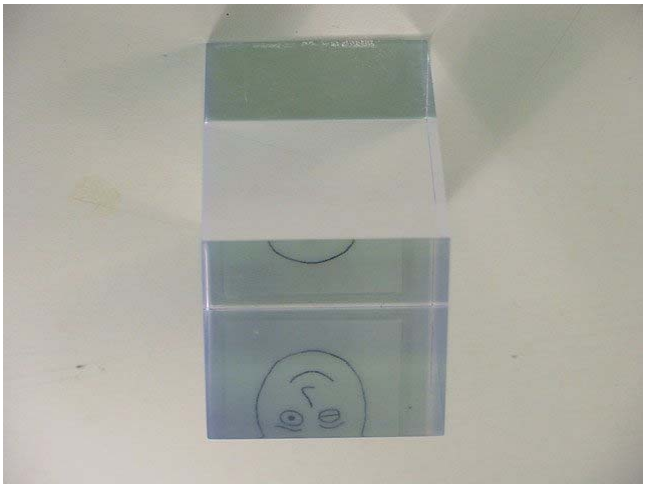
Front: views (6) and (7).



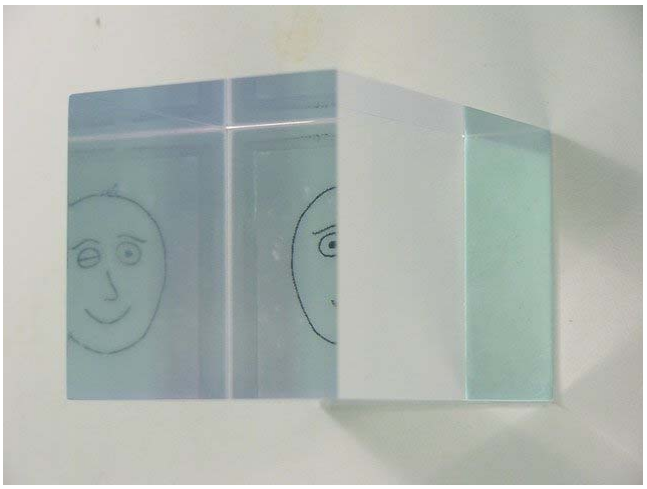
Left: views (2) and (3).



Back: views (8) and (9).



Right: views (4) and (5).



Question #298 is a follow-up to this question.

Archive 15

Question of the Week

Outreach Index Page

Lecture-Demonstration Home Page

For questions and comments regarding the *Question of the Week* contact Dr. Richard E. Berg by e-mail or using phone number or regular mail address given on the Lecture-Demonstration Home Page.

Trick of the Trade

Why Light Won't Refract Through Adjacent Faces of a Cube

John L. Roeder, The Calhoun School, New York, NY

Have you ever tried to refract light through adjacent faces of a cube of glass? The glass blocks we used to get were frosted on alternate sides, so people were discouraged from doing this. But you might be tempted to try it with today's acrylic blocks that have all the sides polished.

Refracting light through adjacent faces of a cube requires the ray pictured in Fig. 1. If n is the index of refraction of the material in the cube relative to the surrounding air, then, by Snell's law,

$$\sin\theta_{i1}/\sin\theta_{r1}' = n \qquad \sin\theta_{i2}/\sin\theta_{r2}' = 1/n.$$

Since $\theta_{i2} = \pi/2 - \theta_{r1}'$,

$$\sin\theta_{i2} = \cos\theta_{r1}',$$

and

$$\begin{aligned}\sin\theta_{r2}' &= n\sin\theta_{i2} = n\cos\theta_{r1}' \\ &= n(1-\sin^2\theta_{r1}')^{1/2} = n(1-\sin^2\theta_{i1}/n^2)^{1/2} \\ &= (n^2-\sin^2\theta_{i1})^{1/2} \le 1,\end{aligned}$$

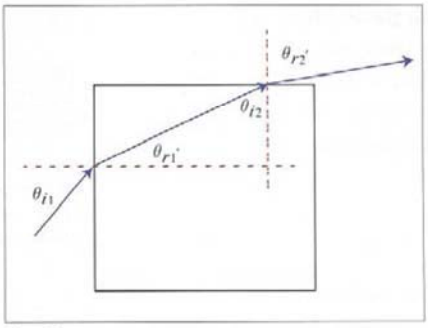


Fig. 1.

since the sine of no angle exceeds 1. Therefore

$$n^2 - \sin^2\theta_{i1} \le 1$$

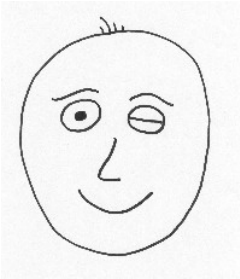
$$n^2 \le 1 + \sin^2\theta_{i1} \le 2$$

$$n \le \sqrt{2}.$$

The index of refraction of typical glasses and plastics is greater than $\sqrt{2}$. On the other hand, the refrative index is less than $\sqrt{2}$ for water. If you have a transparent cubical container with water, try it out.

Background for Face Questions

This week we present one in a series of geometrical optics questions involving the image of the peculiar face below as produced by a variety of optical elements, such as different types of lenses and mirrors. Because there are a large number of them we will do a few at a time and present these problems on alternate weeks, mixed in with problems dealing with other topics.



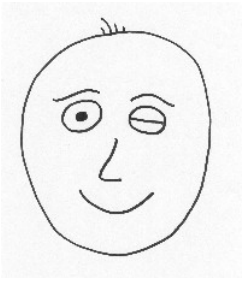
This is a peculiar face, with no ears and only the tiniest lock of hair. No one wanted to call him "Dufus," so we will name him "Otto," because OTTO spelled out in block capital letters reflected in a mirror spells "OTTO."

Below are 28 different possible images of the face above, organized as seven sets of four. These image faces are available for a neat printout on two pages using ".pdf" files: click for Image Faces Page 1 or Image Faces Page 2. Alternatively, these files are available as ".htm" documents: Image Faces Page 1 HTML or Image Faces Page 2 HTML. The document Faces contains all of the faces.

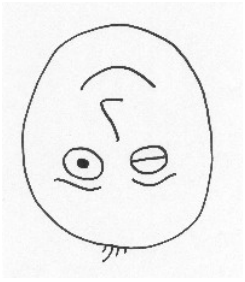
Note that there are seven rows of faces, each of which contains images with four permutations of inversion: normal, inverted vertically, inverted horizontally, and inverted both horizontally and vertically. The seven sets of images show the "normal" face (a-d), the face magnified along both axes (e-h), the face demagnified along both axes (i-l), the face magnified along the vertical axis only (m-p), the face demagnified along the vertical axis only (q-t), the face magnified along the horizontal axis only (u-x), and the face demagnified along the horizontal axis only (y-bb). Note that there is a difference between image sets (m-p) and (y-bb), also between image sets (q-t) and (u-x).

Return to the question for this week by clicking on the "back" button to view the actual questions.

Possible Images



(a)



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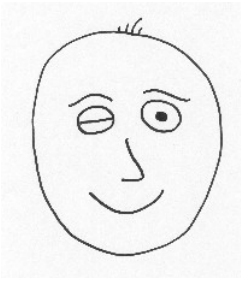
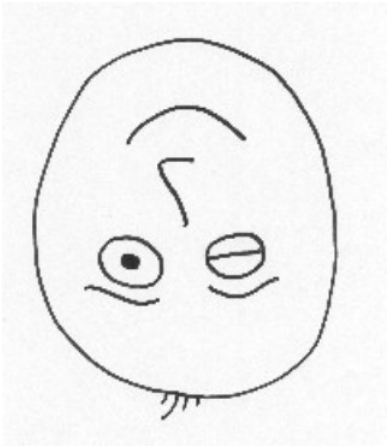


(c)

(e)



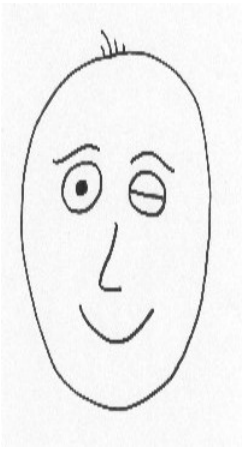
(f)

(g)

(i)

(j)

(k)



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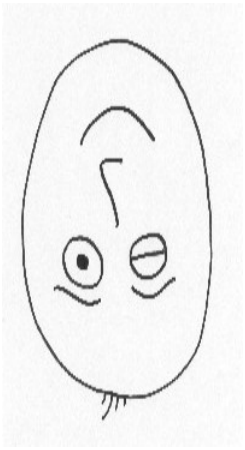

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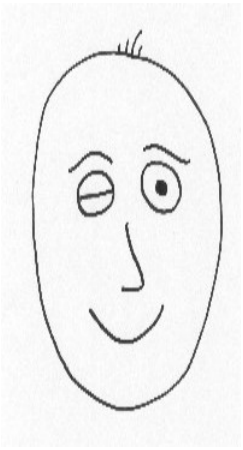
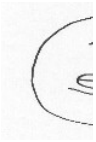


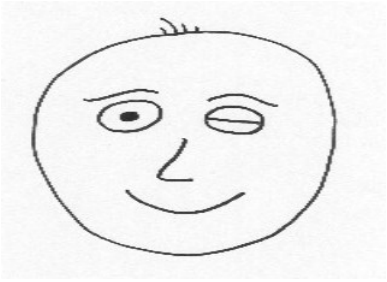
(o)

(q)

(r)

(s)





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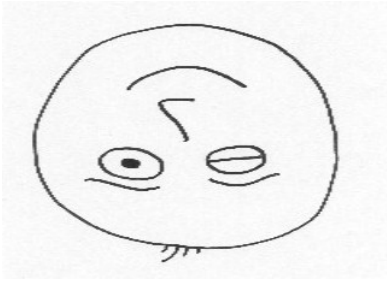

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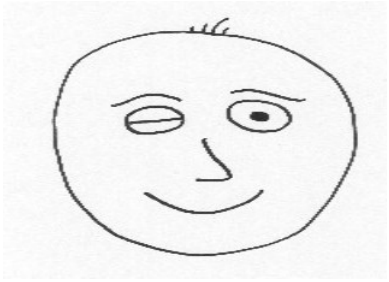

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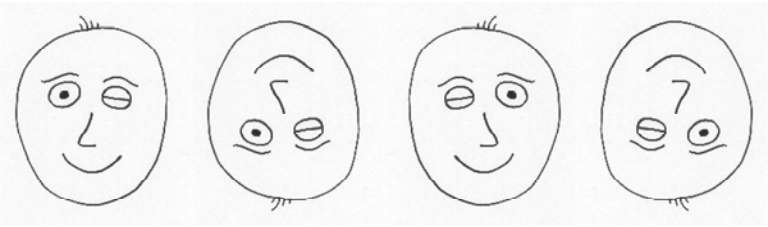

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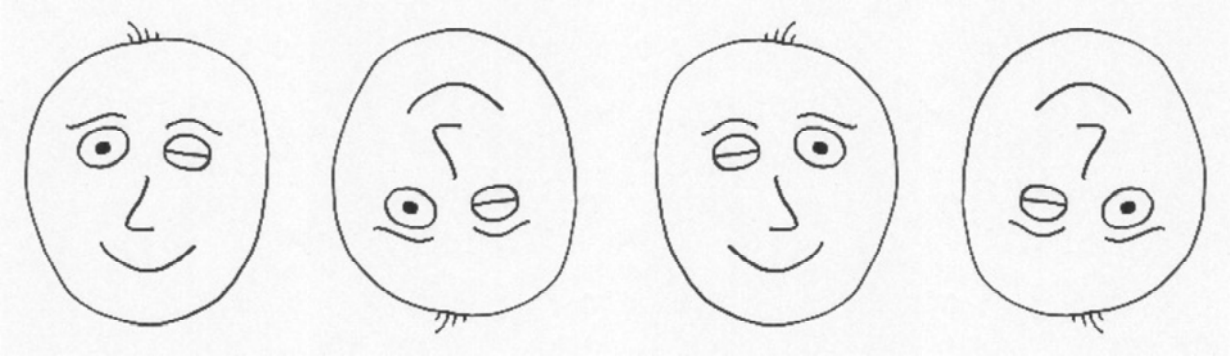

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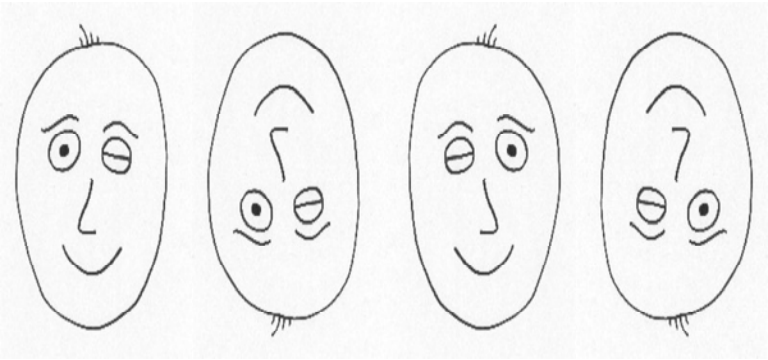

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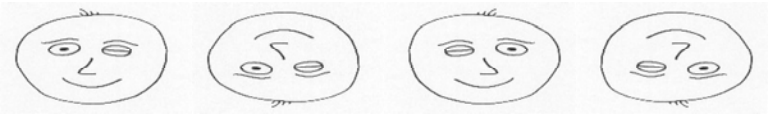

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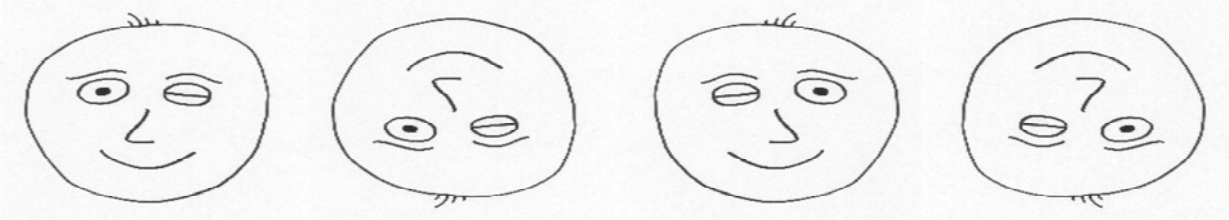

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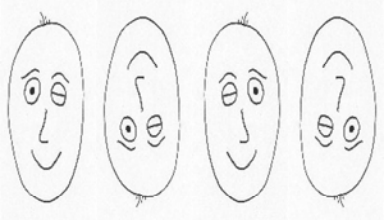

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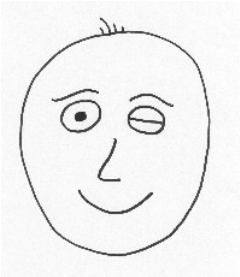
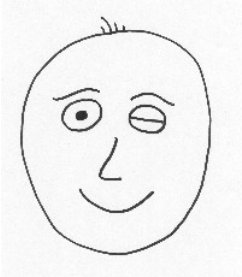


Possible Images



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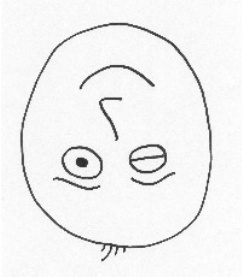

(b)

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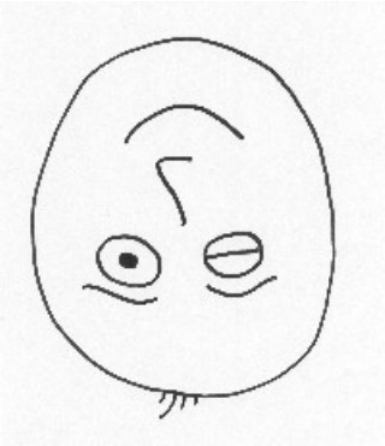

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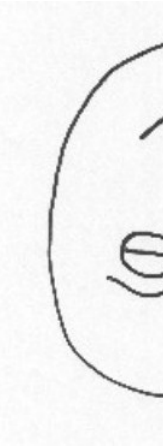

(i)

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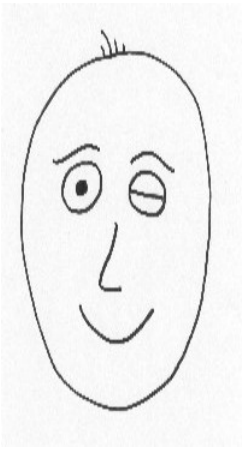

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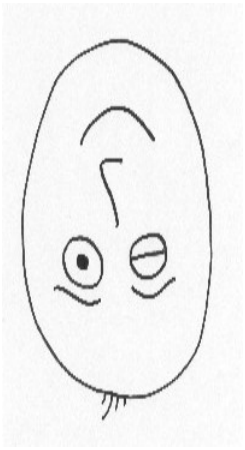

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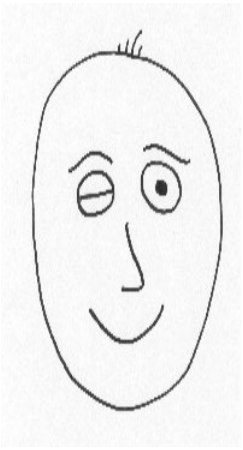

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(q)

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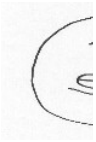

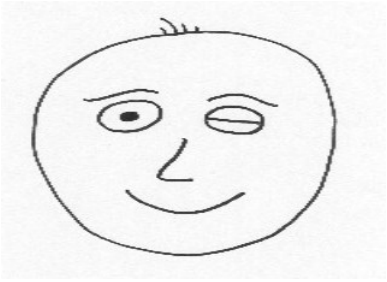

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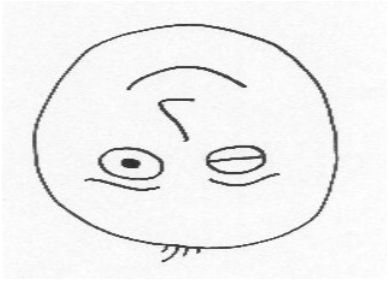

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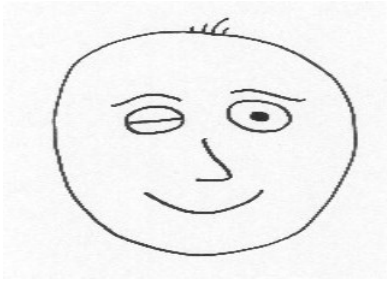

(w)

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