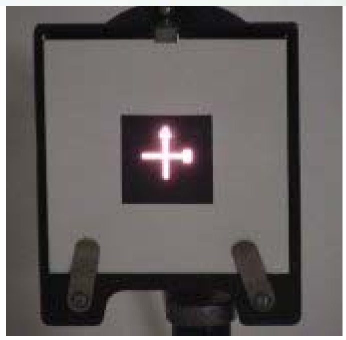
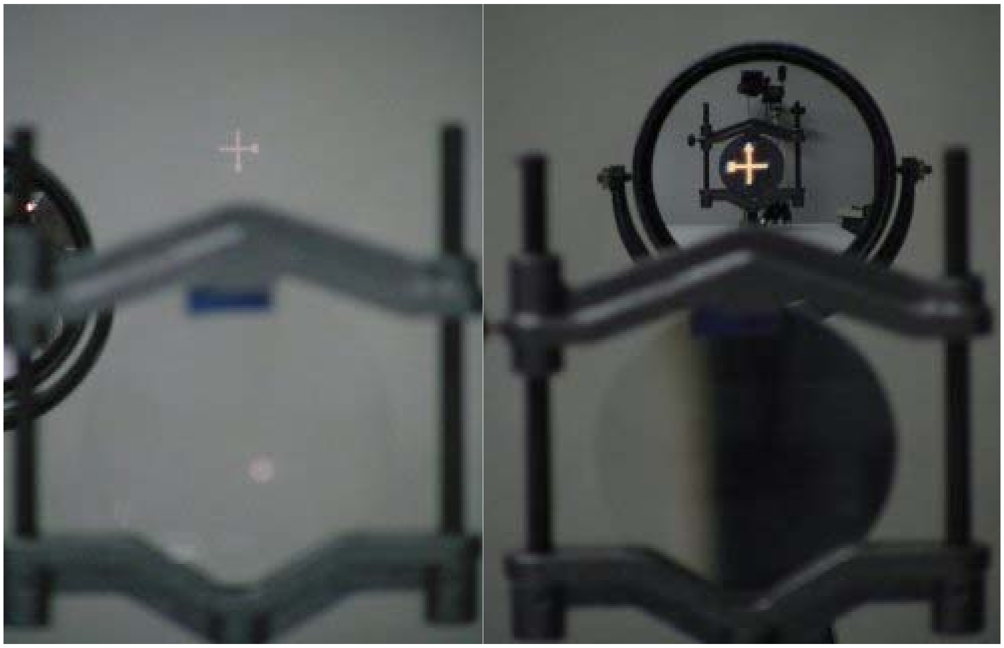
Answer #304

We begin with the two photographs from the question, showing (1) the optical system and (2) looking back along the optic axis of the system at the original object, which we call O_1.

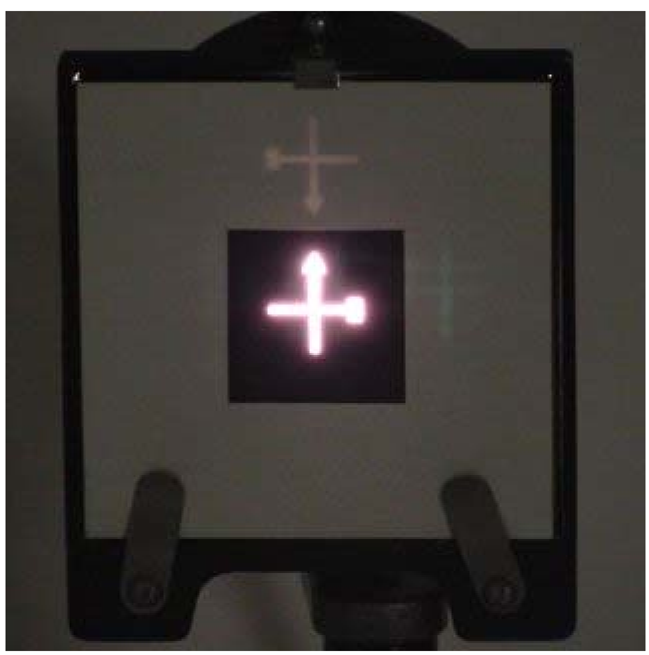


The first image: Image I_1 will be on the opposite side of the lens at the same distance from the lens as the object O_1, will be real, inverted, and have a magnification M=-1 with respect to O_1. It can be observed by removing the mirror and placing a screen at the position of the image. The image is photographed at the left below, looking forward from above the source along the optic axis. Note that the image is in fact inverted; the camera is turned around, looking in the opposite direction. The mirror has been removed and placed at the left of the optic axis (as seen in the photograph).

The second image: The image I_1 serves as the object O_2 for the reflection by the mirror. The mirror then creates an image of object O_2. Because this object is behind the mirror, it is a *virtual* object. In order to photograph this object the mirror was tilted slightly upward so that its optic axis was aimed above the lens. Note that image I_2 is at the same location as O_2 but is inverted with respect to O_2. The magnification M=-1: the image is at the same position as the object, but is inverted.



The third image: The virtual image I_2 of object O_2, behind the mirror, then acts as the object O_3 for the production of final image I_3 by the lens. Because O_2, and thus I_2, are behind the mirror at the same distance as I_1, the lens creates an image I_3 at the same distance to the right of the lens. This is shown in the photograph below. Image I_3 will be inverted with respect object O_3, which makes it inverted with respect to the original object. This can be seen in the photograph below, which compares the final image with the original object. In this photograph the mirror has been tilted slightly upward so that the image is above the original object. As in the case of O_1 and I_1, the magnification M=-1, as seen in the photograph.



For a more detailed analysis, please see set of ray diagrams and concomitant calculations that show the progression from the original object to the final image, available in Microsoft Word or Adobe PDF format.

I used this question in a physics for engineers course some time back. After the students turned in their tests, they were allowed to go behind a curtain and see the actual experiment.

For questions and comments regarding the *Question of the Week* contact
Dr. Richard E. Berg by e-mail or using phone number or regular mail address
given on the Lecture-Demonstration Home Page.

Calculations and ray tracing solution

For these calculations we use the thin-lens object/image/focal length relation:

$$\frac{1}{f} = \frac{1}{Ob} + \frac{1}{Im},$$

Where f is the focal length, Ob is the object distance, and Im is the image distance. The focal length is positive for a convex lens and negative for a convex mirror. A negative image or object distance means that the image or object is virtual: the rays appear to converge toward or diverge from the image or object point, but they are never actually there. For example, the image *or the object* may be behind a mirror. The magnification for any optical element is:

$$M = -\frac{Im}{Ob},$$

where Im is the image distance and Ob is the object distance. Negative magnification indicates an inverted image.

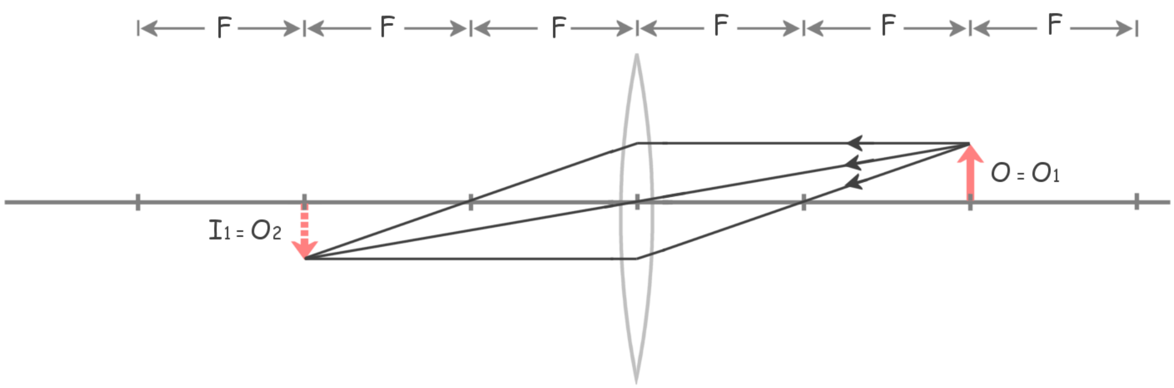
For the current problem, the focal length of the convex lens is $F_L = F$ and that of the convex mirror is $F_M = -F/2$.

To determine I_1 from O_1:

The object is a distance of 2F from the lens, so:

$$\frac{1}{I_1} = \frac{1}{F_L} - \frac{1}{O_1} = \frac{1}{F} - \frac{1}{2F} = \frac{1}{2F}, \quad \text{so } I_1 = 2F \text{ and } M_1 = -\frac{I_1}{O_1} = -\frac{2F}{2F} = -1$$

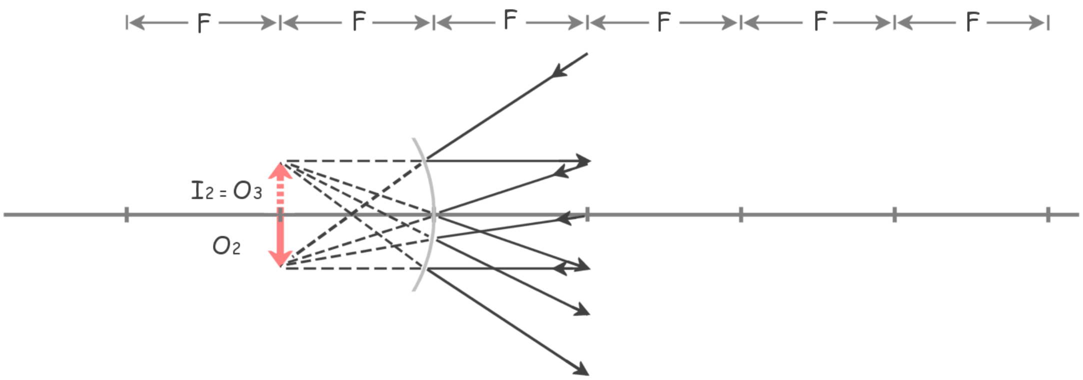
The image is real, and at a distance of 2F on the opposite side of the lens from the object. The magnification is -1, so the image is inverted. This is seen in the ray diagram below:



To determine I_2 from O_2:
When the mirror is inserted the object becomes virtual, that is, it is not really there because the mirror blocks the light rays from getting to its position. This means that the focal length is –F/2 and the object distance is F, so:

$$\frac{1}{I_2} = \frac{1}{F_M} - \frac{1}{O_2} = -\frac{2}{F} - \frac{1}{-F} = -\frac{2}{F} + \frac{1}{F} = -\frac{1}{F}, \text{ so } I_2 = -F \text{ and } M_2 = -\frac{I_2}{O_2} = -\frac{-F}{-F} = -1$$

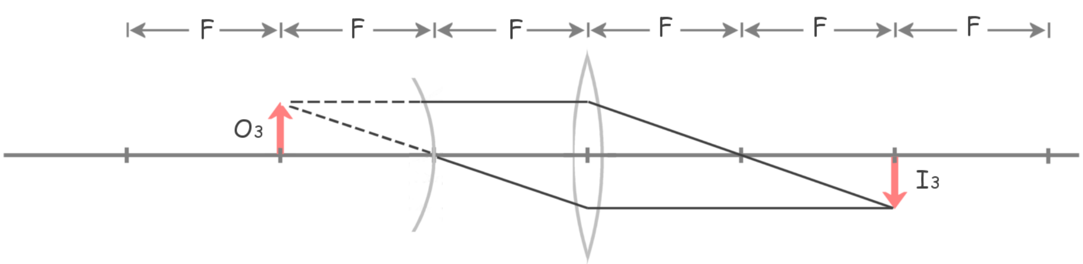
The image is virtual and at a distance of –F behind the mirror. The magnification is -1, so the image I_2 is inverted with respect to O_2, the same orientation as the original object. This is seen in the ray diagram below:



To determine I_3 from O_3:
The third object is behind the mirror a distance of -F, but acts as an actual source of light as the light passes back through the lens. Thus it is a *real* object for this calculation. The final image position is then:

$$\frac{1}{I_3} = \frac{1}{F_L} - \frac{1}{O_3} = \frac{1}{F} - \frac{1}{2F} = \frac{1}{2F}, \text{ so } I_3 = 2F \text{ and } M_3 = -\frac{I_3}{O_3} = -1$$

The image is real, and at a distance of 2F on the opposite side of the lens from the object, the exact location of the original object O_1.. The magnification is -1, so the image is inverted. This is seen in the ray diagram below:



Characteristics of the final image:

The final image is located at the same position as the original object, as seen in the photograph below. Its magnification, $M = M_1M_2M_3 = -1$, so the final image is inverted from the original object, as seen in the photograph.