Answer #310

The answer is (b): the roll in Dan's left hand (freely falling) will reach the floor first, as seen in an mpeg video by clicking your mouse on the photograph below.

Two arguments can be put forward in support of this position, besides that the video clearly shows the roll in free fall reaching the floor first.

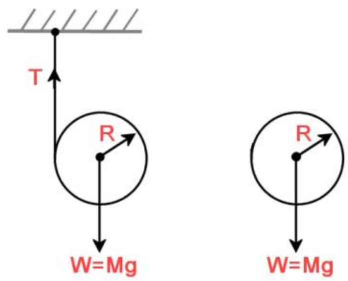
1. For the roll in free fall, the downward acceleration is equal to the force of gravity on the roll divided by its mass: $a_{freefall} = W/m = mg/m = g$, so it accelerates downward with the acceleration of gravity. The downward (gravitational) force on the "unrolling" toilet paper is reduced by the upward tension in the line of toilet paper connecting the roll with Dan's hand: $a_{unroll} = (W-T)/m = g - T/m < a_{freefall}$.

2. After the two rolls have fallen to the floor, the one in free fall will have had (virtually) all of its original gravitational potential energy converted into linear kinetic energy. On the other hand, some of the initial gravitational potential energy of the unrolling toilet paper will have been converted into rotational kinetic energy, leaving less for linear kinetic energy. Because the unrolling roll has less *linear* kinetic energy it must be traveling more slowly and therefore reach the floor later.

Click to see the derivation of equations for this problem as a Word file or as a pdf file.

For questions and comments regarding the *Question of the Week* contact
Dr. Richard E. Berg by e-mail or using phone number or regular mail address
given on the Lecture-Demonstration Home Page.

Equation solution for the toilet paper problem!



The downward acceleration a_f of the freely falling toilet paper roll (f):

$$a_f = \frac{W}{M} = \frac{Mg}{M} = g$$

The downward acceleration a_u of the unrolling toilet paper (u):

$$a_u = \frac{W-T}{M} = g - \frac{T}{M}$$

The equation for angular acceleration of the unrolling toilet paper:

$$\alpha = \frac{\tau}{I} = \frac{R\,T}{\gamma\, M\, R^2} = \frac{T}{\gamma MR}$$

where τ is the torque causing the toilet paper to unroll, I is its moment of inertia, and γ is the ratio of the actual moment of inertia I of the roll to MR^2 (a number between ½ and 1).

The constraint between angular and linear acceleration of the unrolling toilet paper:

$$a_u = \alpha R \quad \text{or} \quad \alpha = \frac{a_u}{R}$$

Combining these equations and eliminating α:

$$a_u = \frac{g}{1+\gamma} < g \quad \text{Q.E.D.}$$