Answer #66

The answers are (b): the scale reading will fall just after time $t=0$, and (d): the scale reading will rise just before $t=T$, as can be seen on an mpeg video by clicking your mouse on the photograph at the left below.

The photograph at the right shows the setup from the side. A metal plate attached to the surface of the scale and extending into a water bath serves as a damping device to limit the excursion and oscillation of the scale after a rapid change occurs. You may notice from the video that we are actually using small lead shot in place of sand to enhance any effects.

During the time when the falling sand forms a complete column between the opening in the bottom of the upper chamber and the mound of sand in the lower container, the weight of sand in the air column, which is the loss in weight of the system, is:

$$dW = \left(\frac{dM}{dt}\right) g T_f,$$

where $T_f$ is the time for the sand to fall, $dM/dt$ is the rate at which the sand is falling, and $g$ is the acceleration of gravity.

On the other hand, as the sand column stops when it hits the mound of sand below it incurs a force equal to the change in momentum of the sand:

$$dF = \frac{dP}{dt} = v \left(\frac{dM}{dt}\right) = \left(\frac{dM}{dt}\right) g T_f,$$

where $v = g T_f$ is the velocity $v$ of the sand column when as it stops.

Using calculus with a bit of trigonometry it can be seen that $dW = dF$, so the scale reading should be very nearly $W$ during the steady-state period when the sand column is complete. On the other hand, when the sand is just beginning to fall but before it reaches the bottom the weight will be slightly less, and after the sand is all out of the upper chamber but before it has all reached the bottom the weight will be slightly more.

*Question of the Week #67* involves exactly what is happening during the steady state situation.
For questions and comments regarding the *Question of the Week* contact [Dr. Richard E. Berg](mailto:dr.berg@umd.edu) by e-mail or using phone number or regular mail address given on the [Lecture-Demonstration Home Page](mailto:lecture-demonstration-home-page).